A Novel Performance Model Given by the Physical Dimensions of Hydraulic Axial Piston Motors : Model Derivation

Heon-Sul Jeong*
Professor, School of Mechanical Engineering, Kunsan National University, Kunsan, Chonbuk 573-701, Korea

Since Wilson (1948) had firstly developed steady-state flow rate and moment loss model for hydraulic piston machines, several authors tried to make precise performance models. However, Huhtala (1997) discussed the strength and weakness of some existing models comparing with measurement data and concluded unfortunately that any model is not accurate enough for wide operating ranges. For a hydraulic axial piston motor of swash plate design with rotating cylinder, new performance formula is derived in this paper through theoretical study on leakages and friction losses of three facing gaps and other non-negligible losses. And efficiency surface of an example motor is estimated and extension of the formula for hydraulic pumps is discussed. A novel feature of the derived model is that all coefficients are given by the physical dimensions of a motor, hence allowing calculation and analysis of the performance of a motor in mind.

Key Words : Leak Flow Loss, Hydro-Mechanical loss, Volumetric, Mechanical and Overall Efficiency, Performance Coefficient Model, Hydraulic Axial Piston Motor

Nomenclature

A_o : Opening area of a piston port
a_P : Piston acceleration $w^2 R_P \tan\alpha \cos\theta$
A_P : Cross-section area of a piston, $\pi d_P^2/4$
A_{VN} : Opening area of the valve port notch
A_{SP} : Side view area of a slipper and piston outside of cylinder block
b_{V1}, b_{B2} : Inner, outer breadth of the valve plate sealing ring
C : Performance coefficient related with leakage
C_d : Discharge coefficient of an orifice
c_V : Breadth of the valve delivery/return port
C_w : Drag coefficient of a cylindrical bar
d_B : Diameter of a piston bore
d_{Br} : Diameter of the support bearing
d_P : Diameter of a piston
e_P : Breadth of the piston port edge
F : Force acting on several parts of a machine
f : Friction coefficient
f_P, h_P : Friction coefficient and gap height between a piston and cylinder hole
h_S : Gap height between slipper and swash plate
h_V : Gap height between valve plate and cylinder
K : Performance coefficient related with moment loss
l : Length of several parts
l_B : Length of a piston bore
l_C : Length of the cylinder block
l_F : Length of piston guide, l_{Fo} or $l_{Fo}+z_P$
l_{Fo} : Length of a piston guide or bushing at ODP
M, M_L : Moment, moment loss produced by several forces
M_2 : Output moment of a motor
m_P : Mass of a piston

* Corresponding Author,
E-mail : hsjeong@kunsan.ac.kr
TEL : +82-63-469-4723; **FAX** : +82-63-496-4727
Professor, School of Mechanical Engineering, Kunsan National University, Kunsan, Chonbuk 573-701, Korea. (Manuscript **Received** May 2, 2006; **Revised** November 16, 2006)

n : Rotational speed of a motor in Hz, $w/2\pi$
p : Piston chamber pressure
p_1 : Inlet pressure of a motor
p_2 : Outlet pressure of a motor
p_e : Pressure inside of a motor enclosure
Δp : Pressure difference, $p-p_e$ or p_1-p_2
Q, Q_L : Flow, leak flow through several parts
r_{Si}, r_{So} : Inner, outer radius of the slipper sealing ring
r_{V1}, r_{V2} : Inner, outer radius of the valve plate inner sealing ring
r_{V3}, r_{V4} : Inner, outer radius of the valve plate outer sealing ring
R_C : Radius of the cylinder block
R_H : Inner radius of a motor housing
R_P : Pitch circle radius of pistons
V_g : Motor geometric displacement, $2A_PR_P \tan\alpha$
v_P : Piston velocity, $wR_P \tan\alpha \sin\theta$
z : Number of pistons in a motor
z_o : Piston number located at delivery port side
z_P : Piston displacement, $R_P \tan\alpha(1-\cos\theta)$
α : Tilting angle of the swash plate
β : Bulk modulus of hydraulic oil
ρ : Density of hydraulic oil
$\Delta\theta_{VN}$: Angle of the valve port notch opened
θ : Angular position of a piston or motor
η_{hm} : Hydro-mechanical efficiency of a motor
η_t : Overall efficiency of a motor
η_v : Volumetric efficiency of a motor
λ_P : Equivalent friction coefficient of a piston
μ : Viscosity of hydraulic oil
ω : Rotational speed of a motor in rad/sec, $2\pi n$

Subscript

aP : Piston acceleration
Br : Support bearing
c : Fluid compressibility
C : Cylinder block
ch : Churning, swirl
CH : Cylinder and motor housing
fP, fS : Friction force on a piston, slipper
o : Miscellaneous
p : Pressure dependent term
P : Piston
pBr : Pressure on bearings
pP : Piston pressure
PP : Piston port
PSP : Piston port, slipper and piston
S : Slipper pad or swash plate
SP : Slipper pad, slipper and piston
V : Valve plate
VN : Valve port control notch
vBr : Velocity of bearings
vP : Piston velocity
$\mu P, \mu S, \mu V$: Viscosity in a piston, slipper, valve plate
ωP : Centrifugal force of a piston
μ : Viscosity dependent term
ρ : Density dependent term

Superscript

$\tilde{}$: Fluctuating value of total flow rate or moment
$\bar{}$: Average value of total flow rate or moment

1. Introduction

Since hydrostatic pumps were appeared and used practically in the world around 1600, modern high operating pressure pumps of bent-axis piston type were developed by Thoma in 1930. In the year 1944 a weight to power ratio of 0.3 kg/kW was already realized with hydraulic pumps for aircraft applications. And in 1950s, axial pumps of swash plate design were asserted successful by themselves on the market (Ivantysyn, 2001). Nowadays, due to the integration of electronics into hydrostatic systems, there are increasing demands for compact and intelligent drive systems allowing transmission of large force, moment or power with high accuracy of motion control. Hydrostatic displacement machines form the heart of such modern electronically controlled hydraulic drive systems.

Several authors had tried to make precise performance models for hydraulic piston machines. Existing models can be grouped into two types. One of them is a kind of polynomial model represented with independent variables such as pressure difference, speed and displacement ratio. The other is a sort of performance coefficient model represented based on the physical nature of losses which is generally given by operating variables such as pressure difference and speed. As an example of performance coefficient model, Schlosser's flow loss model and Wilson's moment loss model for pumps are given below respectively, where ε means displacement setting for variable pumps and C_s, C_{st}, C_f, C_d are coefficients to be determined characterizing the performance of a specific machine.

$$Q_L = \varepsilon V_g n - C_s \frac{V_g \Delta p}{2\mu\pi} - C_{st} V_g^{2/3} \sqrt{\frac{2\Delta p}{\rho}}$$

$$M_L = \varepsilon \frac{V_g \Delta p}{2\pi} + C_f \frac{V_g \Delta p}{2\pi} + C_d \mu V_g n + M_{Lo}$$

Some typical models are well summarized in Huhtala (1997), where the weakness and strength of each model are discussed and compared with measurement data, concluding unfortunately that any model is not accurate enough for wide speed and pressure range of 500～3000 rpm and 20～210 MPa. The reason originates primarily from the complex mechanism having rotating and reciprocating parts with friction working in viscous fluid of hydraulic oil, the physical properties of which is dependent on the operating conditions.

As for the polynomial model, all coefficients of the model are to be determined from measured experimental data by using curve fitting method. And the coefficients of the so-called performance coefficient model are also to be obtained through experiments. Hence, existing models are said to be models for indicating and/or calculating the performance of an already manufactured and tested machine at certain operating point, rather than models for analyzing and evaluating the performance of a designed machine before test or designing machine in advance.

In this article, the performance coefficient model for a hydraulic axial piston motor of swash plate design with rotating cylinder block is considered. This paper is organized as follows: Section 2 describes the working principle of hydraulic axial piston machine and assumptions laid for model derivation. Section 3 and 4 derive formulas for losses of three facing gaps and formulas for other auxiliary losses in a motor are derived in section 5. From the analysis of previous two sections, a new performance model is proposed in section 6 and discussions for an example motor and conclusions are followed in the last section.

2. Working Principle of Axial Piston Motors and Assumptions for Model Derivation

Among two types of axial piston machines such as swash plate and bent axis type, the swash plate design shown in Fig. 1 consists of several components such as piston, slipper, cylinder, swash plate,

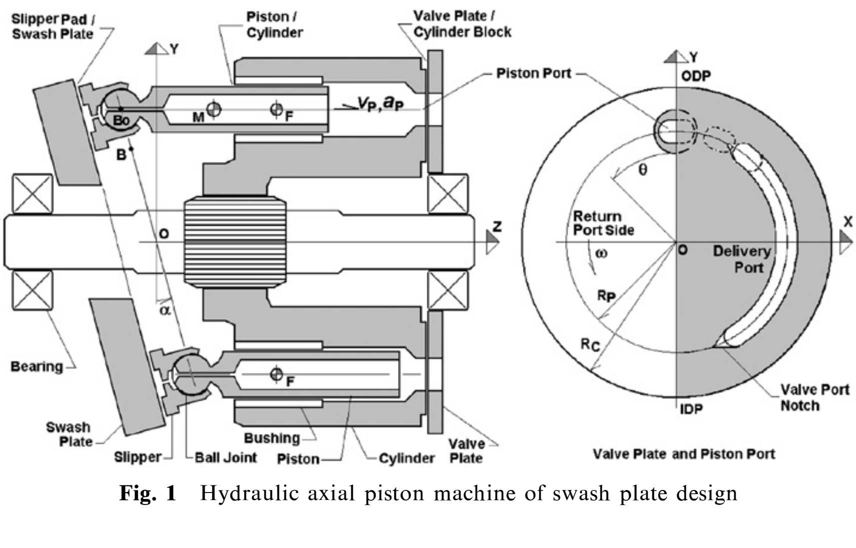

Fig. 1 Hydraulic axial piston machine of swash plate design

valve plate and shaft. According to the kinematic arrangement of rotating and fixed components, swash plate machine is again classified into a design with rotating cylinder and a design with rotating swash plate. Let us briefly describe motoring cycle of the first design, which is main concern of this article.

When hydraulic fluid of high pressure is delivered from external power source into a piston around the inner dead point (IDP) through the delivery port of the valve plate, the piston is forced to move out of cylinder block and then the slipper connected with a ball joint slides on the swash plate. As the result, pistons, the cylinder block and the output shaft are forced to rotate, as shown in Fig. 1. When the piston arrives at the outer dead point (ODP), it is switched onto the return port of the valve plate and then the used fluid of low pressure is returned to external tank during angular period of $0<\theta<\pi$. Hence, the pressure in a piston chamber naturally entails periodic change during a revolution. The cycle of which is depicted in Fig. 2, where the effects of fluid compressibility and viscosity on the piston pressure are also indicated. However, at the instant of switching between the delivery and return port, the pressure in a piston chamber of real machines do not match exactly with that of the motor inlet or outlet. Fluctuating pressure behavior occurring due to such non-ideal piston port switching, especially at the high pressure side of IDP and at the low pressure side of ODP, is neglected in this paper.

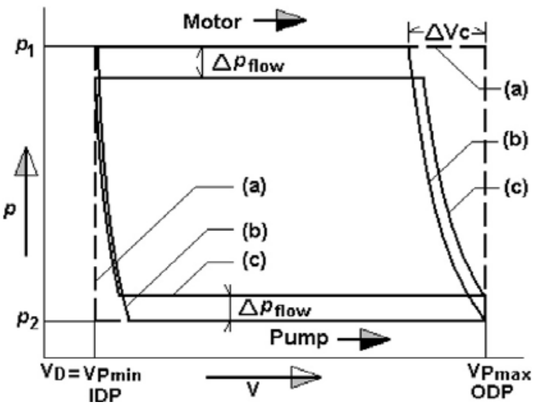

Fig. 2 Indicator diagram of axial piston machine (a) ideal machine (b) fluid compressibility effect (c) fluid viscosity effect

In the following sections, the pressure in a piston chamber is assumed to undergo the ideal process of Fig. 2(a). That is, the piston chamber pressure $p=p_2$ for $0<\theta<\pi$ and $p=p_1$ for $\pi<\theta<2\pi$. Losses due to the effects of fluid compressibility and viscosity are dealt separately. And the pressure of a motor outlet p_2 and inside of a motor enclosure p_e are assumed to be the same.

Meanwhile, as shown in Fig. 1, there exist three facing gaps moving with relative speed between reciprocating and/or rotating members. Leak flows through the gaps and hydro-mechanical friction forces acting on the gaps may degrade the efficiency of a motor. The instantaneous gap heights of three facing parts are to show cyclic behavior per revolution and hence to be functions of angular position θ of the motor. But, since our aim is to formulate the global feature of losses at a certain steady state, all moving gaps are assumed to remain constant over the whole angular range of a revolution. As for the gap between a piston and cylinder block, it is also assumed to stay concentric. Note that, according to Ivantysyn (2001), the computed flow for a tilted piston is almost same to that of concentric ring gap. As for the leakage between the valve plate and cylinder block, the varying breadth b_V and length as well as the height of the corresponding gap are to be accounted for, especially around the IDP and ODP. But their effect on the leakage does not seem to be decisive. Hence, in this view the gap heights are interpreted in the paper as average values of varying heights for one cycle.

3. Flow Rate and Hydro-Mechanical Losses of Three Facing Gaps

Flow rates through three facing gaps are described first in the followings. Flow rate of the fluid contained inside a piston chamber of the cylinder block induced by piston motion can be given

$$Q_{uP}=A_P\cdot v_P=A_P\cdot w\cdot R_P\tan\alpha\cdot\sin\theta \qquad (1)$$

Note that for the angular range of $0<\theta<\pi$ it is a returning flow rate from a motor and for the range $\pi < \theta < 2\pi$ it is a delivered flow rate into a motor. The leak flow through the gap between a piston and cylinder block is

$$Q_{LP}=\frac{\pi d_P \cdot h_P^3}{12\mu \cdot l_F}(p-p_e)-\frac{\pi d_P}{2}\cdot h_P \cdot v_P \tag{2}$$

The first term of Eq. (2) is called as Poiseuille's flow induced by external pressure difference between both sides of a gap. And the second term is so-called Couette flow induced by moving wall of a gap. The leakage flowing out through the piston bore and then through the gap between the slipper and swash plate is

$$Q_{LS}=\frac{\pi h_S^3 \cdot d_B^4}{\mu\left\{6d_B^4 \cdot \ln\frac{r_{So}}{r_{Si}}+128h_S^3 \cdot l_B\right\}}(p-p_e) \tag{3a}$$

$$Q_{LS}=\frac{\pi h_S^3}{6\mu \ln\frac{r_{So}}{r_{Si}}}(p-p_e) \tag{3b}$$

Eqs. (3a) and (3b) correspond to leakage for a piston with capillary type opening of laminar character and orifice type opening of turbulent character, respectively. In case of orifice type opening with reasonable diameter, note that the pressure of recess hole in a slipper is almost same with the piston chamber pressure. Since there are two flow paths on the gap between the cylinder block and valve plate i.e., inner and outer sealing ring of the valve plate as shown in Fig. 3, the leak flow can be approximated assuming parallel fixed gap with breadth λ_V and length b_{V1}, b_{V2} as

$$\begin{aligned} Q_{LV}&=\frac{h_V^3}{12\mu}\cdot(p-p_e)\cdot\left(\frac{\lambda_V}{b_{V1}}+\frac{\lambda_V}{b_{V2}}\right) \\ &\quad+\frac{h_V(b_{V1}+b_{V2})}{2}\cdot R_P\omega, \quad \begin{cases}\lambda_V \simeq (d_P-2e_P)\\ l_V=\dfrac{b_{V1}b_{V2}}{b_{V1}+b_{V2}}\end{cases} \\ &=\frac{h_V^3}{12\mu}\cdot(p-p_e)\cdot\frac{\lambda_V}{l_V}+\frac{h_V(b_{V1}+b_{V2})}{2}\cdot R_P\omega \end{aligned} \tag{4}$$

Basically, there are two types of forces in hydraulic piston machines such as pressure dependent and pressure independent forces. Forces acting on a motor are depicted in Fig. 4. Forces acting on three moving gaps are described in the followings. Pressure force pushing a piston out of the cylinder block in z-direction can be given by

$$F_{pP}=A_P(p-p_e)=\pi d_P^2(p-p_e)/4 \tag{5}$$

The inertia force acting also in z-direction on a piston is

$$\begin{aligned} F_{aP}&=m_P \cdot a_P \\ &=m_P \cdot \omega^2 \cdot R_P \tan\alpha \cdot \cos\theta \end{aligned} \tag{6}$$

The friction force induced by viscous hydraulic oil acting on a piston yields

$$F_{\mu P}=\mu\frac{v_P}{h_P}\cdot \pi d_P \cdot l_F+\frac{\pi d_P \cdot h_P}{2}\cdot(p-p_e) \tag{7}$$

Note that length of piston guide l_F is a constant l_{Fo} for a cylinder with bushing, so-called short piston guide as shown in Fig. 1. But for a cylinder without bushing, it is equal to $l_{Fo}+z_P$ as shown in Fig. 4. And the centrifugal force acting on a

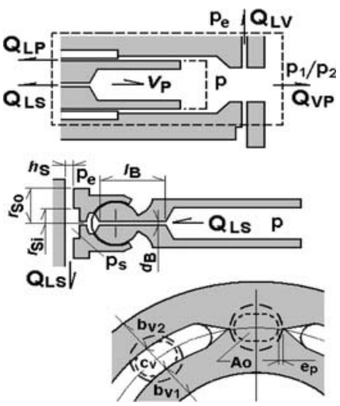

Fig. 3 Detail of three major loss parts

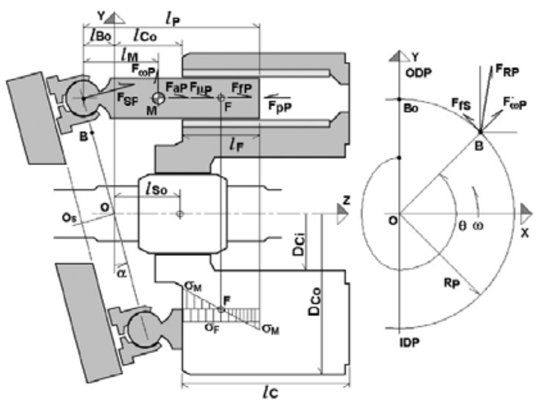

Fig. 4 Forces acting on a piston and slipper

piston in radial direction from the shaft center O can be given by

$$F_{\omega P}=m_P\cdot a_r=m_P\cdot R_P\cdot\omega^2 \tag{8}$$

The frictional force produced on the sliding surface of a slipper acts on a piston in tangential direction. Such a viscous friction can be determined as

$$F_{fS}=\int_{r_{S_i}}^{r_{S_o}}\tau\cdot 2\pi r\cdot dr=\mu\frac{\omega R_P}{h_S}\pi(r_{So}^2-r_{Si}^2) \tag{9}$$

It can be seen from Fig. 4 that magnitude of the reaction force acting onto a piston from the swash plate has the following relation.

$$\begin{aligned}F_{SP}&=(F_{pP}+F_{aP}+F_{\mu P}+F_{fP})/\cos\alpha\\&\equiv F_{AP}/\cos\alpha\end{aligned} \tag{10}$$

Among the above mentioned forces acting on a piston, some of them are out of z-axis tilting the piston and thus generating z-directional friction force. The friction force can be represented as the following equation of Coulumb type, which is proportional to the resultant normal force, F_{RP} exerted on a piston as shown in Fig. 4.

$$\begin{aligned}F_{fP}&=f_{Peq}\cdot F_{RP},\qquad f_{Peq}\equiv f_P\cdot(2l_{Bo}\cos\theta+2l_{Co}+l_F-f_Pd_P\,sign(v_P)/l_F\\&\simeq\frac{f_{Peq}\tan\alpha\,sign_n(v_P)}{1-f_{Peq}\tan\alpha\,sign(v_P)}(F_{pP}+F_{aP}+F_{\mu P})\equiv\lambda_P(F_{pP}+F_{aP}+F_{\mu P}),\ \lambda_P\equiv\frac{A+B\cos\theta}{a+b\cos\theta}\end{aligned} \tag{11a}$$

$$\left.\begin{aligned}&A=f_P\tan\alpha((l_P+l_{Co})\,sign(v_P)-f_Pd_P)\\&B=f_P\tan\alpha l_{Bo}\,sign(v_P)\\&a=(l_P-l_{Co})-f_P\tan\alpha((l_P+l_{Co})\,sign(v_P)-f_Pd_P)\\&b=-(1+f_P\tan\alpha\,sign(v_P))\,l_{Bo}\end{aligned}\right\}:\ \text{w/o bushing}$$

$$\left.\begin{aligned}&A=f_P\tan\alpha((l_{Fo}+2l_{Co})\,sign(v_P)-f_Pd_P)\\&B=2f_P\tan\alpha l_{Bo}\,sign(v_P)\\&a=l_{Fo}-f_P\tan\alpha((l_{Fo}+2l_{Co})\,sign(v_P)-f_Pd_P)\\&b=-2f_P\tan\alpha l_{Bo}\,sign(v_P)\end{aligned}\right\}:\ \text{with bushing} \tag{11b}$$

Note that the parameter λ_P is a function of friction coefficient f_P between a piston and cylinder block. And all the others of λ_P are given by the physical dimensions of a motor. The pressure force acting on a piston makes the slipper slide on the swash plate and in turn makes the cylinder block and motor shaft rotate. Driving moment generated by the pressure force can be given

$$M_{pP}=F_{pP}\cdot R_P\tan\alpha\cdot\sin\theta \tag{12}$$

Meanwhile, moments produced by other forces described until now are loss moments instead of driving moments. Such hydro-mechanical dissipating moments are derived in the followings. Since the centrifugal force $F_{\omega P}$ do not serve in z-directional moment, moment produced due to three useless forces acting on a piston yields

$$\begin{aligned}M_{LP}&=(F_{aP}+F_{\mu P}+F_{fP})\cdot R_P\tan\alpha\cdot\sin\theta\\&\equiv M_{LaP}+M_{L\mu P}+M_{LfP}\end{aligned} \tag{13a}$$

$$\begin{aligned}M_{LfP}&\simeq\lambda_P(F_{pP}+F_{aP}+F_{\mu P})\cdot R_P\tan\alpha\cdot\sin\theta\\&\equiv M_{LfPp}+M_{LfPa}+M_{LfP\mu}\end{aligned} \tag{13b}$$

The loss moment due to viscous friction acting on a slipper is

$$M_{LS}=F_{fS}\cdot R_P \tag{14}$$

Assuming linear pressure distribution in radial direction on the gap between the cylinder block and inner and outer sealing ring of the valve plate, corresponding frictional moment can be obtained as follows (Shuyan, 1998)

$$\begin{aligned}M_{LV}&=\int_{r_{V_1}}^{r_{V_4}}r\cdot\tau\cdot 2\pi r\cdot dr\\&=\mu\frac{\pi}{2h_V}\cdot\omega\cdot(r_{V4}^4-r_{V3}^4+r_{V2}^4-r_{V1}^4)\\&\equiv K_{\mu V}\cdot\mu n\end{aligned} \tag{15}$$

4. Average Flow Rates and Moments Behaviour of Three Moving Gaps

All the flow rates, forces and hydro-mechanical moments discussed in the previous section are formulas represented at a certain instantaneous angular position θ of a piston. Generally several number of pistons are arranged in the cylinder block of a motor at equiangular positions with same pitch circular radius. Hence, the averages of total flow rates and moments for a motor with pistons of number z are derived in this section. The term "average" used hereafter without any special comment in this article means the average value over one revolution period of total sum of force or moment for each piston.

Total delivered flow rate into a motor through the delivery port of the valve plate, which will be the same as total returning flow rate out of the motor through the return port, induced by the reciprocating piston motion can be given from Eq. (1) as

$$\tilde{Q}_{vP}=\sum_{i=1}^{z_o}A_P\cdot v_P=\sum_{i=1}^{z_o}A_P\cdot w\cdot R_P\tan\alpha\cdot\sin\theta_i \cong\sum_{i=1}^{z}A_P\cdot w\cdot R_P\tan\alpha\sin\theta_i\cdot sign(p_i) \quad (16)$$

where z_o means the number of pistons located at the delivery port side and θ_i stands for the angular position of a piston. The piston velocity induced flow rate $\tilde{Q}_{vP}$ fluctuates naturally with fixed frequency, which depends on the motor speed. Average value of the fluctuating flow rate yields

$$\bar{Q}_{vP}=wA_PR_P\tan\alpha\cdot\frac{z}{\pi}\equiv V_g\cdot\frac{w}{2\pi}=V_g\cdot n,\quad n=\frac{w}{2\pi} \quad (17)$$

where V_g is generally called as geometric displacement of a motor. Detail derivation of Eq. (17) is given in appendix. For the leak flow rate though the piston gap, note that the Poiseuille flow in Eq. (2) leaks out from the delivered flow over half revolution of a piston. And, because average for sum of half period sine curves is z/π average of total Couette flow of a motor becomes as follows. Hence, average leak flow rate through the piston gap can be determined as

$$\bar{Q}_{LP}=\left\{\begin{array}{ll}(p-p_e)\frac{\pi d_P h_P^3}{12\mu l_{Fo}}\cdot\frac{z}{2} & \text{with bushing}\\ (p-p_e)\frac{\pi d_P h_P^3}{12\mu}\cdot\frac{z}{2\sqrt{a^2-b^2}} & \text{w/o bushing}\end{array}\right\},\quad \left\{\begin{array}{l}a=l_{Fo}+R_P\tan\alpha\\ b=R_P\tan\alpha\end{array}\right. + \frac{\pi d_P h_P}{2}\cdot\omega\cdot R_P\tan\alpha\cdot\frac{z}{\pi}\equiv C_{\mu P}\frac{\Delta p}{\mu}+C_{vP}n \quad (18)$$

Since leak flows through the gaps of slippers and the valve plate occurs only for half period of $\pi<\theta<2\pi$ during one revolution. Hence, corresponding mean values for the flow rates are

$$\bar{Q}_{LS}=\left\{\begin{array}{ll}\frac{\pi d_B^4 h_S^3}{\mu\{6d_B^4\ln r_{So}/r_{Si}+128h_S^3 l_B\}}\cdot\frac{z}{2}\cdot(p-P_e) & \text{capillary type}\\ \frac{\pi h_S^3}{6\mu\ln r_{So}/r_{Si}}\cdot\frac{z}{2}\cdot(p-p_e) & \text{orifice type}\end{array}\right\} \equiv C_{\mu S}\frac{\Delta p}{\mu} \quad (19)$$

$$\bar{Q}_{LV}=(p-p_e)\cdot\frac{h_V^3}{12\mu}\frac{\lambda_V}{l_V}\cdot\frac{z}{2}+\frac{h_V(b_{V1}+b_{V2})}{2}\cdot R_P\omega\cdot\frac{z}{2} \equiv C_{\mu V}\frac{\Delta p}{\mu}+C_{vV}n \quad (20)$$

Average of the fluctuating motor driving moment generated by the pressure force can be determined by

$$\bar{M}_{pP}=F_{pP}R_P\tan\alpha\cdot\frac{z}{\pi} =\Delta pA_P\cdot R_P\tan\alpha\cdot\frac{z}{\pi}\equiv V_g\frac{\Delta p}{2\pi} \quad (21)$$

Among three loss moment contributions in Eq. (13), average moment $\bar{M}_{aP}$ due to piston inertia force is zero, since it consists of pure sinusoidal terms with period 2π and its mean over the period is zero. Hence, viscous frictional moment $M_{\mu P}$ and Coulumb-like frictional moment M_{LfP} are loss components consumed by reciprocating pistons and average of fluctuating instantaneous moment $\tilde{M}_{LP}$ for a motor becomes

$$\bar{M}_{LP}=\bar{M}_{L\mu P}+\bar{M}_{LfP}\simeq\bar{M}_{L\mu P}+(\bar{M}_{LfPp}+\bar{M}_{LfPa}+\bar{M}_{LfP\mu}) \approx\bar{M}_{L\mu P}+M_{LfPp} \quad (22a)$$

$$\bar{M}_{L\mu P}=\left\{\begin{array}{ll}\mu\frac{\pi d_P}{h_P}\cdot(R_P\tan\alpha)^2 l_{Fo}\cdot w\cdot\frac{z}{2} & \text{with bushing}\\ \mu\frac{\pi d_P}{h_P}\cdot(R_P\tan\alpha)^2(l_{Fo}+R_P\tan\alpha)\cdot w\cdot\frac{z}{2} & \text{w/o bushing}\end{array}\right\} +\frac{\pi d_P\cdot h_P}{2}\cdot R_P\tan\alpha\cdot\Delta p\cdot\frac{z}{\pi}\equiv K_{\mu Pv}\cdot\mu n+K_{\mu Pf}\Delta p \quad (22b)$$

$$\bar{M}_{LfPp}\simeq|\bar{\lambda}_P|\cdot\Delta pA_PR_P\tan\alpha\cdot\frac{z}{\pi}\equiv K_{fPf}\cdot\Delta p \quad (22c)$$

$$\bar{M}_{LfPa}\simeq\bar{\lambda}_P^{\pm}\cdot m_P(R_P\tan\alpha)^2\cdot w^2\cdot\frac{11z}{12}\equiv K_{fPa}\cdot n^2 \quad (22d)$$

$$\bar{M}_{LfP\mu} \simeq \begin{cases} \bar{\lambda}_P^{\pm} \cdot \mu \dfrac{\pi d_P}{h_P} \cdot (R_P \tan\alpha)^2 l_{Fo} \cdot w \cdot \dfrac{z}{5} & \text{with bushing} \\ \bar{\lambda}_P^{\pm} \cdot \mu \dfrac{\pi d_P}{h_P} \cdot (R_P \tan\alpha)^2 \left(l_{Fo} + \dfrac{5}{7} R_P \tan\alpha \right) \cdot w \cdot \dfrac{z}{5} & \text{w/o bushing} \end{cases}$$
$$+ |\bar{\lambda}_P^{-}| \frac{\pi d_P \cdot h_P}{2} \cdot R_P \tan\alpha \cdot \Delta p \cdot \frac{z}{\pi} \equiv K_{fP\mu\nu} \cdot n + K_{fP\mu p} \cdot \Delta p \tag{22e}$$

$$\bar{\lambda}_P^{-} = \left(\frac{B^-}{b^-} + \frac{A^- b^- - a^- B^-}{b^- \sqrt{(a^-)^2 - (b^-)^2}} \right),$$
$$\bar{\lambda}_P^{+} = \left(\frac{B^+}{b^+} + \frac{A^+ b^+ - a^+ B^+}{b + \sqrt{(a^+)^2 - (b^+)^2}} \right), \tag{22f}$$
$$\bar{\lambda}_P^{\pm} \equiv \bar{\lambda}_P^{+} + \bar{\lambda}_P^{-}$$

where $\bar{M}_{LfPa}$ and $\bar{M}_{LfP\mu}$ are neglected simply because magnitude of them are generally very small compared with that of M_{LfPp}. Parameters A^+, B^+, a^+, b^+ and A^-, B^-, a^-, b^- are values of A, B, a, b defined in Eq. (11b) when $sign(v_P)$ is replaced by $+1$ and -1, respectively. As for the meaning of the last term z/π in Eq. (22), refer to appendix A.2. Total moment dissipated due to the slippers friction can be given as

$$\bar{M}_{LS} = \sum_{i=0}^{z} F_{fS} \cdot R_P = z F_{fS} R_F$$
$$= z \cdot \mu \frac{\omega R_P}{h_S} \pi (r_{So}^2 - r_{Si}^2) \cdot R_p \equiv K_{\mu S} \cdot \mu n \tag{23}$$

5. Other Losses of a Hydraulic Piston Motor

Besides losses of three facing parts, other volumetric and hydro-mechanical losses which can not be neglected exist. It will be described in this section. Up to here, the piston pressure is assumed to undergo the ideal process shown in Fig. 2. In other words, the piston pressure p on delivery post side is assumed equal to the pressure p_1 supplied into a motor. However, as mentioned in section 2, the actual piston pressure is slightly less than that because of the pressure loss, which occurs when flow passes through the piston port with opening area A_o. The pressure loss of turbulent character affects on the performance of a motor. Such effects on the loss of flow rate and moment can be represented as follows

$$M_{LPP} = 2 \cdot \frac{V_g}{2\pi} \cdot z \Delta p_{flowPP} = \frac{V_g^3}{\pi z V_{dPP}^2 A_o^2} \rho n^2$$
$$\equiv K_{PP} \cdot \rho n^2 \tag{24}$$

$$Q_{LPP} = 2 \cdot \bar{Q}_{vP} \frac{\Delta p_{flowPP}}{\Delta p} = \frac{\rho V_g^3}{z^2 C_{dPP}^2 A_o^2} \frac{n^3}{\Delta p}$$
$$\equiv C_{PP} \frac{\rho n^3}{\Delta p} \tag{25}$$

Detail of the reasoning for the above two equations is described in appendix A.4. Generally one or two valve notches so-called kidney, with quite small area A_{VN} are provided. It is intensionally inserted in order to reduce the pressure fluctuation occurring at the trailing end of the valve ports because of the non-ideal piston port switching. The leak flow rate of turbulent character through the valve control notch and its equivalent moment loss can be described as follows

$$Q_{LVN} = z \cdot \frac{\Delta\theta_{VN}}{2\pi} \cdot Q_{VN} = \frac{z \Delta\theta_{VN} C_{dVN} A_{VN}}{2\pi} \sqrt{\frac{\Delta p}{\rho}}$$
$$\equiv C_{VN} \sqrt{\frac{\Delta p}{\rho}} \tag{26}$$

$$M_{LVN} = \frac{V_g}{2\pi} \Delta p_{eqVN} = \frac{z \Delta\theta_{VN} C_{dVN} A_{VN}}{(2\pi)^2} \frac{\Delta p \sqrt{\Delta p / \rho}}{n}$$
$$\equiv K_{VN} \frac{\Delta p \sqrt{\Delta p / p}}{n} \tag{27}$$

where $\Delta\theta_{VN}$ stands for the angular range of the valve notch. When two valve notches are provided on a port in the valve plate, leak flow and loss moment of Eqs. (26) and (27) must be doubled. Another source of volumetric loss during a motoring cycle originates from fluid compressibility as shown in Fig. 2. From the reduction of effective volume ΔV_c of the hydraulic fluid (Ivantysyn, 2001), the equivalent leak flow rate can be described as

$$Q_{Lc} = z \Delta V_c n = z \cdot V_{P\max} \frac{\Delta p}{\beta} \cdot n = z V_{P\max} \frac{\Delta p}{\beta_{oAF} / \xi_\beta} n$$
$$\equiv C_c \cdot \xi_\beta n \Delta p \tag{28}$$

where $V_{P\max}$ means the largest volume of a piston chamber at ODP which is sum of the smallest volume, so-called dead volume $V_D = V_{P\min}$ and the volume swept during a piston stroke. Meanwhile, slippers and pistons outside of the cylinder block as well as the cylinder block itself rotate inside a motor enclosure at the same speed with the motor, which is filled with hydraulic oil. Such drag loss of slippers and pistons and loss of concentric cylinder block, so-called churning losses can be formulated as follows

$$\begin{aligned} M_{LchSP} &= \sum_{i=1}^{z} F_w R_P = z \cdot C_{wSP} \frac{\rho}{2} A_{SP} R_P^2 w^2 \cdot R_P \\ &\equiv K_{SP} \cdot \rho n^2 \end{aligned} \tag{29}$$

$$M_{LchCH} = \pi R_C l_C (R_H + R_C) \cdot \mu w \equiv K_{CH} \cdot \mu n \tag{30}$$

On the other hand, generally two bearings are installed before and after the cylinder block in order to support the block against the forces acting on the motor shaft. Dissipated moments according to the applied normal force on the bearings and according to the rotating speed of the bearings are described

$$\begin{aligned} M_{LBr} &\simeq f_{vBr} \cdot 160 \cdot d_{Br}^3 + f_{vBr} \cdot (60\mu/\rho)^{2/3} n \cdot d_{Br}^3 \\ &\quad + f_{pBr} \cdot \frac{z}{2}(1+\bar{\lambda}_P) \cdot A_P \Delta p \tan\alpha \cdot d_{Br} \\ &\equiv M_{LoBr} + K_{\mu Br} \cdot \mu n + K_{pBr} \cdot \Delta p \end{aligned} \tag{31}$$

6. Performance Model of a Hydraulic Piston Motor

In the previous sections, leak flow rates and loss moments of three facing parts and other non-negligible losses are derived. One can find that all of them are represented in two operating variables of a motor such as pressure difference Δp and rotating speed n. And the flow rate to be delivered into a motor $\bar{Q}_1$ is the flow rate $\bar{Q}_{vP}$ induced by the piston motion plus the sum of all leakages $\bar{Q}_L$ mentioned until now. Therefore, followings are readily given

$$\bar{Q}_1 = \bar{Q}_{vP} + \bar{Q}_L = V_g n + \bar{Q}_L \tag{32}$$

$$\begin{aligned} \bar{Q}_L &= (\bar{Q}_{LP} + \bar{Q}_{LS} + \bar{Q}_{LV}) + Q_{LVN} + Q_{LPP} + Q_{Lc} + Q_{Lo} \\ &= (C_{\mu P} + C_{\mu S} + C_{\mu V}) \frac{\Delta p}{\mu} + (C_{vP} + C_{vV}) n \\ &\quad + C_{VN} \sqrt{\frac{\Delta p}{\rho}} + C_{PP} \frac{\rho n^3}{\Delta p} + C_c \xi_\beta n \Delta p + Q_{Lo} \\ &= C_{\mu PSV} \frac{\Delta p}{\mu} + C_{vPV} n + C_{VN} \sqrt{\frac{\Delta p}{\rho}} + C_{PP} \frac{\rho n^3}{\Delta p} + C_c \xi_\beta n \Delta p + Q_{Lo} \\ &\equiv C_\mu \Delta p + C_v n + C_{\rho VN} \sqrt{\Delta p} + C_{\rho PP} \frac{n^3}{\Delta p} + C_{\beta c} n \Delta p + Q_{Lo} \end{aligned} \tag{33}$$

At the last line of Eq. (33), leak flows of same character are grouped together. The last term Q_{Lo} in Eq. (33) is not a physical leakage but a term introduced for compensating error of flow-meters used in measurements (Ivantysyn, 2001). The output moment generated by a motor $\bar{M}_2$ is the moment $\bar{M}_{pP}$ produced by the pressure force minus the sum of all loss moments $\bar{M}_L$ mentioned until now. Therefore, one can get readily

$$\bar{M}_2 = \bar{M}_{pP} - \bar{M}_L = V_g \Delta p / 2\pi - \bar{M}_L \tag{34}$$

$$\begin{aligned} \bar{M}_L &= M_{LP} + (\bar{M}_{LS} + M_{LV} + M_{LchCH} + M_{L\mu Br}) \\ &\quad + M_{LpBr} + (M_{LPP} + M_{LchSP}) \\ &\quad + M_{LVN} + (\bar{M}_{L\mu P} + \bar{M}_{LfPp}) + M_{Lp2} + (M_{LoBr} + M_{LoAux}) \\ &= (K_{\mu Pv} + K_{\mu S} + K_{\mu V} + K_{CH} + K_{\mu Br}) \mu n \\ &\quad + (K_{\mu Pp} + K_{fPp} + K_{pBr}) \Delta p + (K_{PP} + K_{SP}) \rho n^2 \\ &\quad + K_{VN} \frac{\Delta p}{n} \sqrt{\frac{\Delta p}{\rho}} + K_{p2} \Delta p^2 + (M_{LoBr} + M_{LoAux}) \\ &\equiv K_\mu n + K_p \Delta p + K_{\rho PSP} n^2 + K_{\rho VN} \Delta p \sqrt{\Delta p} / n \\ &\quad + K_{p2} \Delta p^2 + M_{Lo} \end{aligned} \tag{35}$$

where loss moments of same character are grouped together at the last line. M_{Lp2} in Eq. (35) stands for the pressure dependent moment loss due to mixed and/or boundary friction which may happen especially at extremely high pressure. And M_{LoAux} is a term accounting for the neglected constant moment losses due to for example pre-loading of seals and springs, etc. in addition to the measurement error of the sensor used. Note that the output moment $\bar{M}_2$ of Eq. (34) becomes negative at very low pressure, because at such case the driving moment $\bar{M}_{pP}$ proportional to the pressure Δp becomes smaller than the total loss moment $\bar{M}_L$. It means that a motor can not rotate at very low pressure.

Volumetric efficiency and hydro-mechanical efficiency of a motor are generally defined as $\eta_v = n V_g / \bar{Q}_1$ and $\eta_{hm} = \bar{M}_2 / (\Delta p V_g / 2\pi)$, respectively. And normally accepted definition of overall efficiency for a motor is the product of the volumetric efficiency and hydro-mechanical efficiency as

$$\begin{aligned} \eta_t = \eta_v \cdot \eta_{hm} &= \frac{n V_g}{\bar{Q}_1} \cdot \frac{2\pi \bar{M}_2}{\Delta p V_g} = \frac{\bar{M}_2 \cdot w}{\bar{Q}_1 \cdot \Delta p} \\ &= \frac{(\bar{M}_{pP} - \bar{M}_L) \cdot w}{(\bar{Q}_{vP} + \bar{Q}_L) \cdot \Delta p} \end{aligned} \tag{36}$$

Following the definition, new formula for the overall efficiency of a motor yields finally

$$\eta_t = \frac{2\pi n}{\Delta p} \frac{V_g \Delta p / 2\pi - (K_\mu n + K_p \Delta p + K_{\rho PSP} n^2 + K_{\rho VN} \Delta p \sqrt{\Delta p} / n + K_{p2} \Delta p^2 + M_{Lo})}{V_g n + (C_\mu \Delta p + C_v n + C_{\rho VN} \sqrt{\Delta p} + C_{\rho PP} n^2 / \Delta p + C_c n \Delta p + Q_{Lo})} \tag{37}$$

Note that every coefficients in Eq. (37) standing for the physical nature are defined in Eqs. (33) and (35), which are again composed of several coefficients. And those definitions are also given in sections 4 and 5, As the result, differently with existing models, it should be mentioned that all performance coefficients, except some hard-to-modelling terms such as virtual leakage Q_{Lo}, mixed friction coefficient K_{p2} and auxiliary moment loss M_{LoAux}, in the newly formulated model are represented by the physical dimensions of a motor.

7. Discussions and Conclusions

For an example motor, some principal dimensions of which are given in Table 1, volumetric, hydromechanical and overall efficiencies are calculated by using the new performance model with given dimensional data and plotted in Figs. 5 and 6.

Meanwhile, the state of hydraulic oil can be characterized by three parameters such as density ρ, viscosity μ and bulk modulus β. First two of them are known to be dependent on the operating conditions such as pressure and temperature. The effect of such hydraulic oil property on the efficiency can also be easily included in the newly derived model. Figs. 5 and 6 are graphs reflecting such effects.

Comparing with existing models mentioned in introduction section, the derived flow loss model of Eq. (33) and moment loss model of Eq. (35) includes additional higher order nonlinear terms. Generally the contour curves for the efficiency of real hydraulic motors are not necessarily elliptic.

Table 1 Principal dimension of a hydraulic motor

displacement	V_g	200 cc/rev
tilting angle	α	15 degree
piston number	z	9 ea
piston diameter	d_P	27.9 mm
pitch circle radius	R_P	68.0 mm
piston gap	h_P	19.6 μm
piston shoe gap	h_S	15.2 μm
valve plate gap	h_V	11.8 μm
piston friction coeff.	f_P	0.10

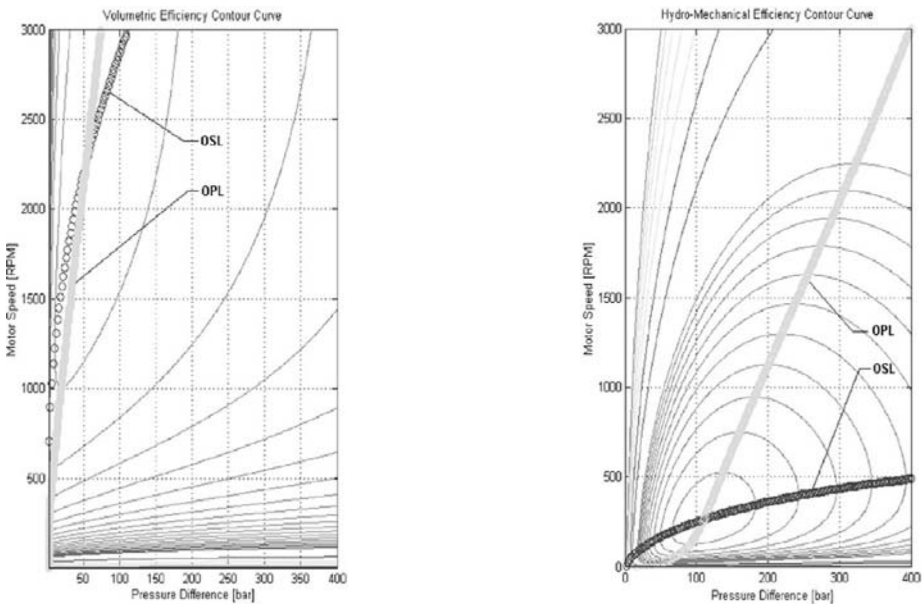

Fig. 5 Volumetric and hydro-mechanical efficiency

Performance model of a quadratic form can represent only elliptic contour curve. But, as show in Figs. 5 and 6, the proposed model can represent distorted elliptical contour curve, which clearly shows the cross-coupling effect between operating speed and pressure.

On the other hand, at very low operating speed viscous friction acting on a facing gap is known to show nonlinear behaviour, that is so-called Stribeck effect (Ivantysyn, 2001). In order to account for the effect on the performance model, corresponding viscosity factor μ in moment loss terms of Eq. (35) may be modified as a speed dependent factor.

In Figs. 5 and 6, an optimal pressure line (OPL) constructed by optimal pressure points with maximum efficiency at a certain speed and an optimal speed line (OSL) constructed by optimal speed points with maximum efficiency at a certain pressure are plotted together. Obviously, the cross point of OPL and OSL lines is the optimal operating point showing best efficiency obtainable by a motor. As for the example motor, one can find from the figures that volumetric efficiency is sensitive to the operating speed, whereas hydro-mechanical and overall efficiencies are dependent on both of the operating pressure and speed of the motor. It accords with well-known trends for the efficiency of hydraulic motors.

As mentioned above, performance of the example motor is calculated with data of the physical dimension of the motor and the property of hydraulic oil. Hence, the formula presented in this article can be an useful tool for estimating optimal operating point of a designed motor in mind and for analyzing the effect of a certain parameter on the overall performance. As for the Rexroth A10VM motor of displacement 45 cc/rev without piston guide bushing, analysis and usage of the new proposed model comparing calculated efficiency with measurement data will be presented in the companion paper, Jeong 2007. In the paper, it is shown that maximum and average efficiency estimation error over operating ranges of speed 300～2700 RPM and pressure 20～300 bar are just 2.33% and within 0.30%, respectively.

The performance model formulated here for hydraulic axial piston motors can be extended to that for pumps. For such extension, some facts should be considered such that hydraulic oil of low pressure is sucked though the valve port in a pump for angular range $0<\theta<\pi$ and high pres-

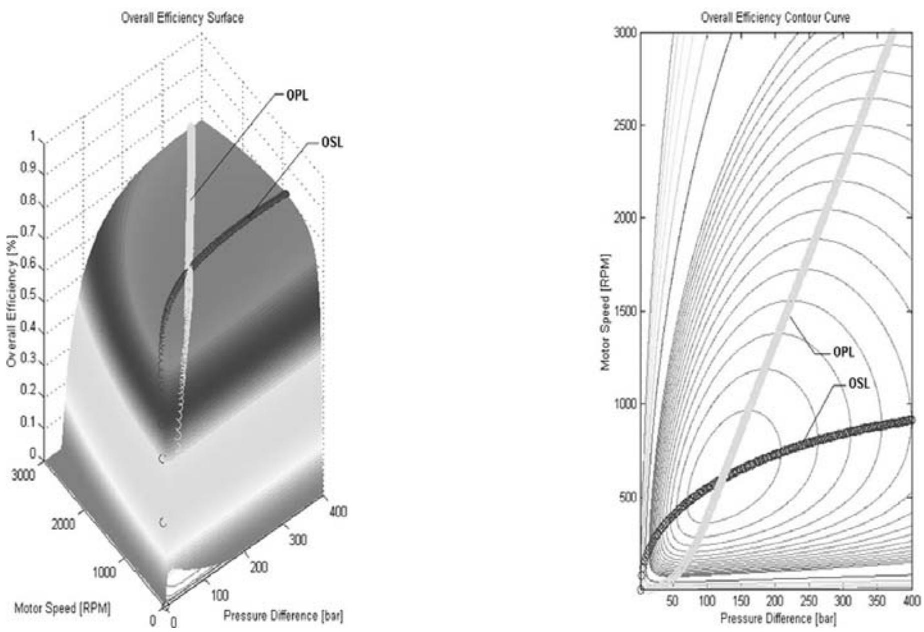

Fig. 6 Overall efficiency of the example axial piston motor

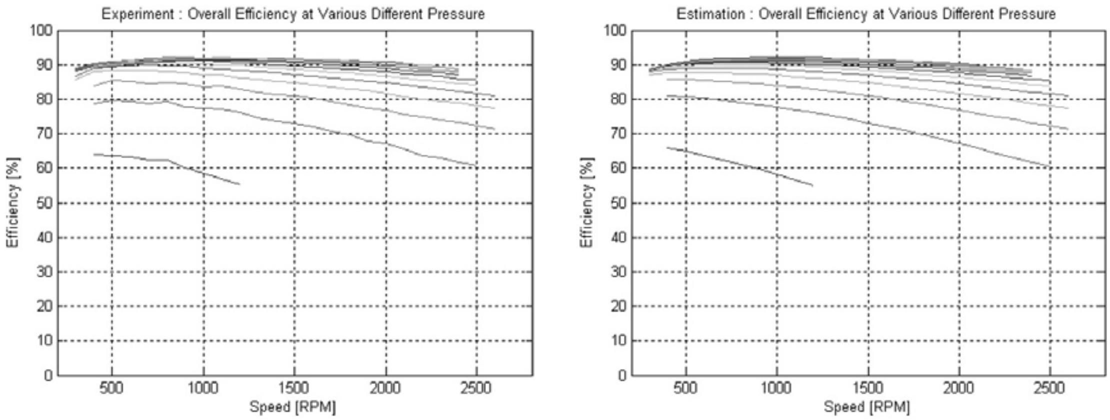

Fig. 7 Measured and estimated overall efficiency of a Rexroth motor

sure oil is delivered for $\pi<\theta<2\pi$. And during the cyclic behaviour shown in Fig. 2(c), the pressure in a piston chamber affected by fluid viscosity is higher than the delivered outlet pressure p_1 for $\pi<\theta<2\pi$ and lower than the inlet pressure p_2 of the suction port. Meanwhile, suction port of a pump especially rotating at high speed may fall into a cavitation condition resulting in reduced suction flow rate, so-called filling loss. Hence filling loss should be considered in the model for a pump, which seems significant at high operating speed.

Appendix

A.1 Piston friction force and moment, Eqs. (11), (13) and (22)

Detail for the derivation procedure of Eqs. (11) and (22) is described in the article of Jeong (2004). And exact and approximate piston friction forces for the motor mentioned in section 7 are depicted in Fig. A1, where the exact and approximate piston friction means frictions calculated by using the first and second equations in Eq. (11a). Friction moment M_{LfP} of a piston in Eq. (13), fluctuating moment $\widetilde{M}_{LP}$ of all pistons and average loss moment $\bar{M}_{LP}$ of Eq. (22) are also shown in Fig. A1.

Fig. A1 shows that the approximated force and moments are quite accurate even though they give slightly smaller value due to effects of neglected forces F_{wP} and F_{fS}.

A.2 Average flow rate, Eq. (17) and average driving moment, Eq. (21)

The number of pistons z_o located at the delivery port side of a motor periodically changes with the angular position θ of a datum piston. It can be arranged as

$$z_o=\begin{cases} z/2 & \text{when } 0<\theta<2\pi/z \quad : \text{for even } z \\ (z+1)/2 & \text{when } 0<\theta<\pi/z \\ (z-1)/2 & \text{when } \pi/z<\theta<2\pi/z \end{cases} \Bigg\} : \text{for odd } z \qquad (A1)$$

Assuming the piston chamber pressure $p_i=0$ for $0<\theta<\pi$ and utilizing the formula for summation of trigonometric functions (Gradshteyn, 2000), sum of the fluctuating flow rate $\widetilde{Q}_{vP}$ can be given as

$$\widetilde{Q}_{vV}=A_P\cdot w\cdot R_P\tan\alpha\cdot\sin\frac{z_o\pi}{z}\sin\left(\theta+\frac{z_o-1}{z}\pi\right)/\sin\frac{\pi}{z}$$
$$=\begin{cases} A_P\cdot w\cdot R_P\tan\alpha\cdot\cos\left(\theta-\frac{\pi}{z}\right)/\sin\frac{\pi}{z}, & \text{for even } z,\ 0<\theta<\frac{2\pi}{z} \\ A_P\cdot w\cdot R_P\tan\alpha\cdot\cos\left(\theta-\frac{\pi}{2z}\right)/\left(2\sin\frac{\pi}{2z}\right), & \text{for odd } z,\ 0<\theta<\frac{\pi}{z} \end{cases} \qquad (A2)$$

By integrating each trigonometric term in Eq. (A2) over one period, one can find its mean value as z/π. Hence, average of the piston velocity induced flow rate yields Eq. (17). Note that the term z/π stands for the summation $\sum_{i=1}^{z}\sin\theta_i\cdot sign(p_i)$ i.e. equally spaced non-negative half-sinusoidal functions for $0<\theta_i<\pi$ or $\pi<\theta_i<2\pi$ for pumps or motors, respectively. On the other hand, in-

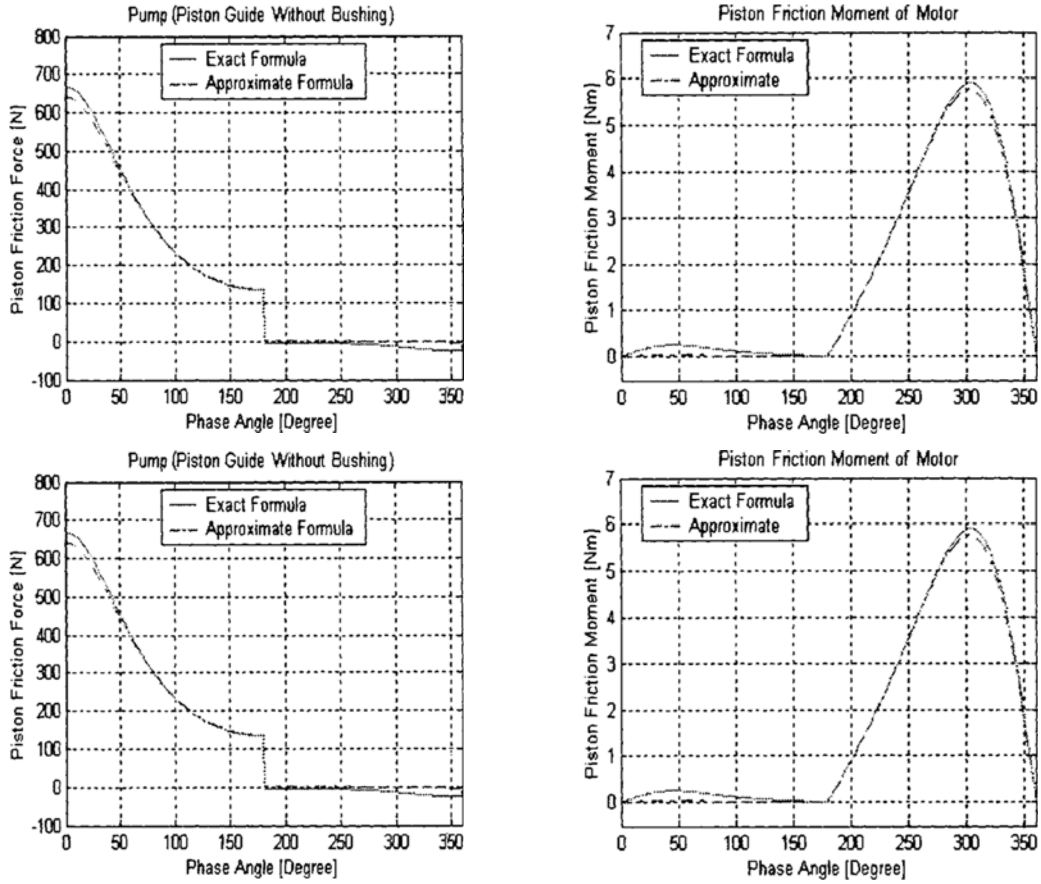

Fig. A1 Piston friction force, fluctuating and average moment

stantaneous driving moment by a motor can be given from Eq. (12) as

$$\tilde{M}_{pP}=\sum_{i=1}^{z_o}F_{pP}\cdot R_P\tan\alpha\cdot\sin\theta_i \qquad \text{(A3)}$$

By using similar treatment applied for derivation of the average delivery flow rate $\bar{Q}_{vP}$, one can get Eq. (21).

A.3 Average flow rate, first term of Eq. (18)

For the case of cylinder block with piston guide bushing, length of the piston guide l_F is the length of the bushing itself l_{Fo}. Hence, Poiseuille leak flow occurs during only half of the one revolution period. Hence, the last term $z/2$ of the first line of Eq. (18) makes sense. But for the case of cylinder block without guide bushing, total leak flow though the piston gaps can be given from Eq. (2) as

$$\begin{aligned}\tilde{Q}_{LP}&=\sum_{i=1}^{z}(p-p_e)\frac{\pi d_P\cdot h_P^3}{12\mu\cdot(l_{Fo}+z_{Pi})}\\&=(p-p_e)\frac{\pi d_P\cdot h_P^3}{12\mu}\cdot\sum_{i=1}^{z}\frac{sign(p-p_e)}{l_{Fo}+z_{Pi}}\end{aligned} \qquad \text{(A4)}$$

The last term of Eq. (A4) is a trigonometric function in a form $1/(a-b\cos\theta)$. By utilizing the integral relationship of such a function (Gradshteyn, 2000) for $a^2>b^2$, one can get

$$\begin{aligned}\int_{\pi}^{2\pi}\frac{d\theta}{a-b\cos\theta}&=\int_{0}^{\pi}\frac{d\theta}{a+b\cos\theta}\\&=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\frac{\sqrt{a^2-b^2}\tan\theta/2}{a+b}\Bigg|_0^{\pi}\\&=\frac{\pi}{\sqrt{a^2-b^2}}\end{aligned} \qquad \text{(A5)}$$

and the average of (A5) over motoring period $\pi<\theta<2\pi$ is $1/\sqrt{a^2-b^2}$. Hence, multiplying the average value and a term $z/2$ reflecting the fact that pressure is applied only for half revolution period, one can finally obtain average leak flow through piston gaps, first term of Eq. (18). Accuracy of Eq. (18) is verified by numerical calculation for both even and odd number of pistons.

A.4 Piston port equivalent loss, Eqs. (24) and (25) and valve notch equivalent loss, Eq. (27)

Macroscopically considering a motor as a machine consuming flow rate $\bar{Q}_{vP}$ through piston ports with area A_o and number z, flow velocity v_{flowPP} through and its equivalent pressure drop Δp_{flowPP} across a piston port can be approximated by

$$\begin{aligned} v_{flowPP} &= \bar{Q}_{vP}/(zA_o) \\ \Delta p_{flowPP} &= \rho v_{flowPP}^2/(2C_{dPP}^2) \end{aligned} \tag{A6}$$

Also consider the fact that a hydraulic motor is a machine converting hydraulic energy delivered by high pressure oil into mechanical energy rotating output shaft with torque. And the relationship between the delivered pressure and output torque is given by Eq. (21). Hence, the pressure drop in Eq. (A6) can be given as an equivalent moment loss Eq. (24). Note that the multiplication factor 2 in Eqs. (24) and (25) means that there occur pressure drops twice in one motoring cycle, as show in Fig. 2. By equating energy loss due to pressure drop with nominal flow rate $\bar{Q}_{vP}$ and energy loss due to leak flow with nominal pressure Δp as

$$2\Delta p_{flowP}\bar{Q}_{vP} = Q_{LPP}\Delta p \tag{A7}$$

an equivalent leak flow Q_{Lpp} due to pressure drop across the piston ports can be given as Eq. (25).

By applying the reasoning for loss of piston ports onto the valve control notch, one can get equivalent pressure drop Δp_{eqVN} across the valve control notch that is related with leak flow Q_{LVN} through the valve notch as follows

$$Q_{LVN}\Delta p = \bar{Q}_{vP}\Delta p_{eqVN} \tag{A8}$$

Hence, the corresponding moment loss due to the pressure drop Δp_{eqVN} of valve notch yields Eq. (27).

A.5 Churning loss, Eqs. (29) and (30)

Drag force of cylindrical bar-like slippers and pistons outside of the cylinder block with side-view area A_{SP} moving in a fluid with linear velocity $v_{SP} = R_P \cdot w$ is

$$F_w = c_{wSP} \cdot \frac{\rho}{2} \cdot A_{SP} \cdot v_{SP}^2 \tag{A9}$$

Hence the churning moment loss due to rotating slippers and pistons yields Eq. (29). Meanwhile, assuming linear velocity profile $\partial u/\partial r = R_C w/(R_H - R_C)$ along the radial gap between inner surface of the motor housing and outer surface of the cylinder block, one can obtain the churning loss moment Eq. (30), by integrating shear stress of Newtonian fluid $\tau = \mu \cdot \partial u/\partial r$ over the circumferential area $A_C = 2\pi r \cdot l_C \cdot dr$ of the cylinder block over the moment arm r from R_C to R_H.

A.6 Bearing loss, Eq. (31)

According to Jang 1997, loss moment of a bearing dissipated by its rotating speed and applied normal force is given by

$$M_{LBr} = M_{LvBr} + M_{LpBr} \tag{A10a}$$

$$M_{LvBr} = \begin{cases} M_{LoBr} = f_{vBr} \cdot 160 \cdot d_{Br}^3 \\ \qquad (\mu/\rho \cdot n < 1/30000) \\ M_{L\mu Br} = f_{vBR} \cdot (60\mu n/\rho)^{2/3} \cdot d_{Br}^3 \\ \qquad (\mu/\rho \cdot n > 1/30000) \end{cases} \tag{A10b}$$

$$M_{LpBr} = f_{pBr} \cdot \bar{F}_{nBr} \cdot d_{Br} \tag{A10c}$$

where the normal force $\bar{F}_{nBr}$ applied onto the two bearings is sum of the normal force $F_{SP}\sin\alpha$ acting by each slipper via the cylinder block. From Eqs. (10) and (11), it can be approximated by neglecting inertia and viscous friction forces as

$$\begin{aligned} \bar{F}_{nBr} &\cong \frac{z}{2} \cdot (\bar{F}_{pP} + F_{fP}) \cdot \tan\alpha \\ &\cong \frac{z}{2}(1 + \bar{\lambda}_P) \cdot A_P \Delta p \cdot \tan\alpha \end{aligned} \tag{A11}$$

where the term $z/2$ is inserted because of the statement in the first paragraph of appendix A.3. Hence, one can obtain Eq. (31) where the speed dependent moment of Eq. (A10b) is simplified.

References

Dal-Sik Jang, 1997, *Verlustanalyse an Axialkolbeneinheiten*, Dissertation, Aachen, ISBN 3-89653-252-9.

Gradshteyn, I. S. and Ryzhik, I. M., 2000, *Table of Integrals*, Series, and Products, 6th Ed., Academic Press.

Heon-Sul Jeong, Hyoung-Eui Kim, Bo-Sik Kang, Dong-Soo Jeong and Young-Bum Lee, 2003, "Preliminary Design of Hydraulic Piston Motors for Achieving an User-Defined Optimal Efficiency point," *1st Int. Conf. on Computational Methods in Fluid Power Technology*, Melbourne Autralia, pp. 613～624.

Heon-Sul Jeong and Hyung-Eui Kim, 2004, "On the Instantaneous and Average Piston Friction of Swash Plate Type Hydraulic Axial Piston Machines," *KSME International Journal*, Vol. 18, No. 10, pp. 1700～1711.

Heon-Sul Jeong, 2006, "A Novel Performance Model Given by the Physical Dimensions of Hydraulic Axial Piston Motors : Experimental Analysis," *to appear in Journal of Mechanical Science and Technology.*

Jaroslav Ivantysyn and Monika Ivantysynova, 2001, HydroStatic Pumps and Motors, Akademia Books International, ISBN 81-85522-16-2.

Kalevi Huhtala, 1997, "Matti Vilenius, Comparison of Steady State Models of Hydraulic Pump," *5th Scandinavian International Conference on Fluid Power*, Linkoping, Sweden.

Shuyan Liu and Weige Yan, 1998, "Analytical Solution for Laminar Viscous Flow in the Gap Between Parallel Rotary Disks," *Journal of Beijing Institute of Technology*, Vol. 7, No. 2, pp. 113～119.

Wilson, W. E., 1948, "Performance Criteria for Positive Displacement Pumps and Fluid Motors," *ASME Semi-annual Meeting*, paper No. 48-SA-14.